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Jean-Marie Husson, Joseph Gril, Frédéric Dubois, Nicolas Sauvat

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J.M. Husson[1], J. Gril[2], F. Dubois[1], N. Sauvat[1]

[1] *Groupe d'Etude des Matériaux Hétérogènes, Université de Limoges, Centre universitaire de Génie Civil, Egletons, France, frederic.dubois@unilim.fr*

[2] *Laboratoire de Mécanique et Génie Civil, Université Montpellier, France, jgril@lmgc.univ-montp2.fr*

Wood is a porous, hygroscopic, anisotropic and non homogeneous biopolymer. It is classified as a viscoelastic material with mechanical properties depending on temperature and moisture content. The effect of creep is an important factor for the design and the durability of timber structures. Creep evolution results from the interaction between mechanical stress and moisture content variations. In this context, Gril [1] proposed a formulation based on “hygro-locks”, a combination of rheological elements with their activation depending on the humidity level. Although this model accounted for several features of the mechano-soptive creep (the so-called ++, - and + effects) it failed to produce a realistic hygromechanical response for other type of loading, such as stress relaxation. Inspired by this concept, Husson [2] proposed a new formulation based on hygro-locks introduced in the elastic response, using the following hereditary form characterized by a time synchronization between mechanical loading and moisture-dependent rigidity:

$$\varepsilon(t) = \int_{0^-}^{t} J_{\max}(t', t) \frac{\partial \sigma}{\partial t'} \, dt'$$

where $J_{max}(\tau,t)$ designates the maximum of elastic compliance during the interval $[\tau,t]$. The resulting stress-strain relationship defines a hygromechanical response represented by a “mechanosorptive spring”. An alternative description, directly based on the hygro-lock concept, is based on a discretisation (u_1 … u_N) of the moisture content. The stress σ and strain ε of the spring are given by $\sigma=\sigma_1$ +… σ_N, $\varepsilon=J_1\sigma_1$ +… $+J_N\sigma_N$, where $J_1<\ldots<J_N$ is a moisture-dependent compliance When the moisture content equals u_n, any stress change is supported by the link n: $d\sigma=d\sigma_n$, so that the instantaneous response of the spring is given by $d\varepsilon=J_n d\sigma$. When the moisture content increases from u_n to u_{n+1}, the stress is transferred from link n to link n+1: $\delta\sigma_{n+1}=\sigma_n$, $\delta\sigma_n.=-\sigma_n$, and the strain increase resulting from this internal load transfer is $\delta\varepsilon=(J_{n+1}-J_n)\sigma_n$. This internal variable formulation is equivalent to the integral formulation when the continuous moisture-dependency J(u) is approximated by a discrete description. With both formulations it is possible to develop a thermodynamic approach based on the first principle, in order to put in evidence the capacity to store energy during drying phases.

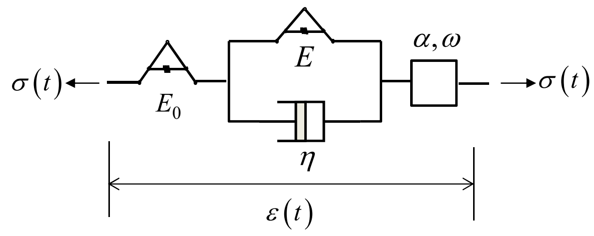

Figure 1: A mechano-sorptive Kelvin Voigt model

A similar hygro activation can be introduced in the viscoelastic behavior represented by a generalized Kelvin-Voigt or Maxwell model, where each spring is replaced by a mechanosorptive spring. These rheological models can be completed by hygro-expansion elements driven by moisture content level and expansion coefficient α. The case of a Kelvin-Voigt link is illustrated in Figure 1, where the springs have been replaced by a symbol of the mechanosorptive spring evoking the hygro-locks. Using the integral formulation, the total strain can be expressed as follow:

$$\varepsilon(t) = \int_0^t J(t',t)\frac{\partial \sigma}{\partial t'}dt' + \int_0^t \alpha \frac{\partial u}{\partial t'}dt'$$

where $J(\tau,t)$ is a creep function given by:

$$J(t',t) = J^0_{\max}(t',t) + \int_{t'}^{t} \frac{1}{\eta(t'')}\exp\left[-\int_{t''}^{t}\frac{dt'''}{J^1_{\max}(t',t''')\eta(t''')}\right]dt''$$

where $J^0=1/E^0$ and $J^1=1/E$ are the instantanzous and delayed compliance, respectively, and η the viscosity coefficient of the dashpot. An incremental formulation of these equations has been successfully applied to predict the 1D response for mechanosorptive tests in L direction. However, the complex time-dependent relationship between stress, strain and moisture content, requires a formulation adapted to finite element implementation. A major obstacle inherent to the integral formulation is the need to store the whole loading history at each integration point. The use of the formulation based on internal variables should allow to overcome this obstacle. The time resolution allows a coupling with a heat and mass transfer algorithm and complex loading with time-dependent boundary conditions.

References

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